# Battery Operations in Electricity Markets

## Strategic Behavior and Distortions

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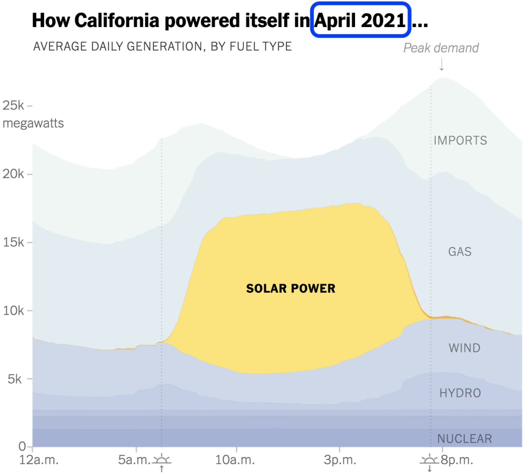
with Santiago R. Balseiro, Omar Besbes, and Bolun Xu

# Motivation

# Battery storage is becoming big enough to move markets




Grid-scale batteries are no longer negligible price takers.

Large batteries can shift energy across hours and across settlements.

Their timing and quantity choices can affect market prices.

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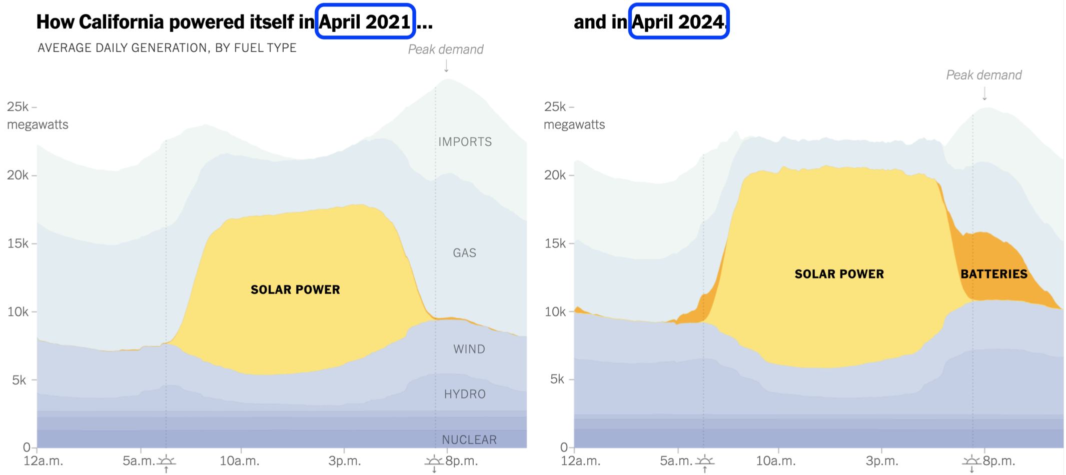


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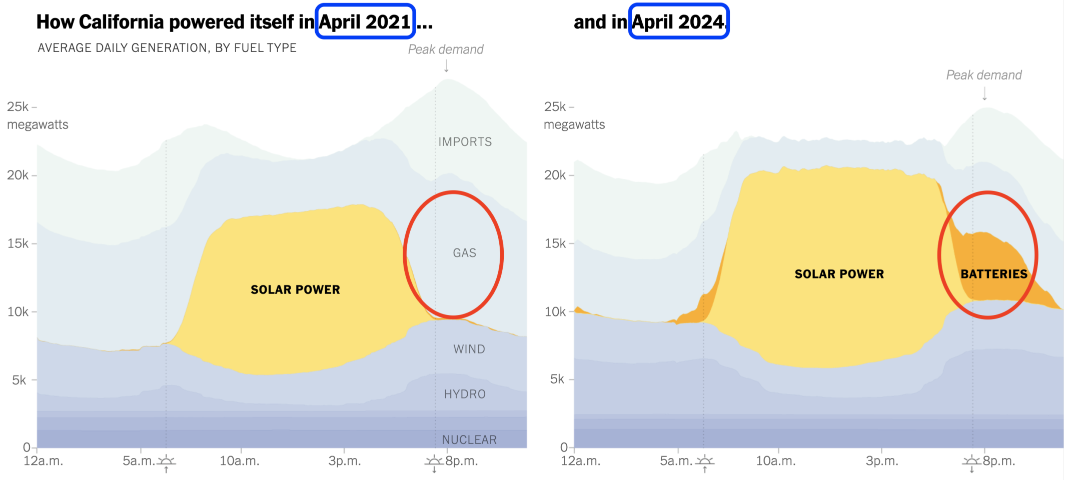


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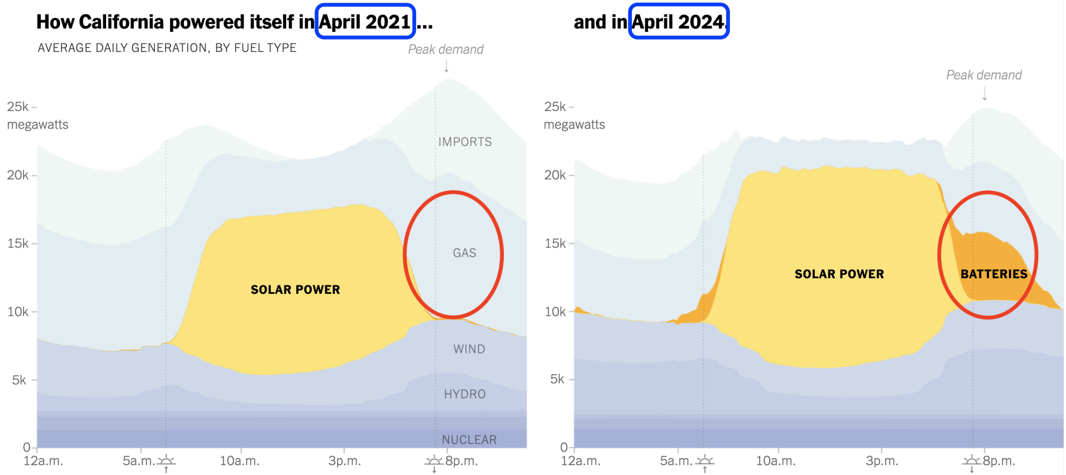


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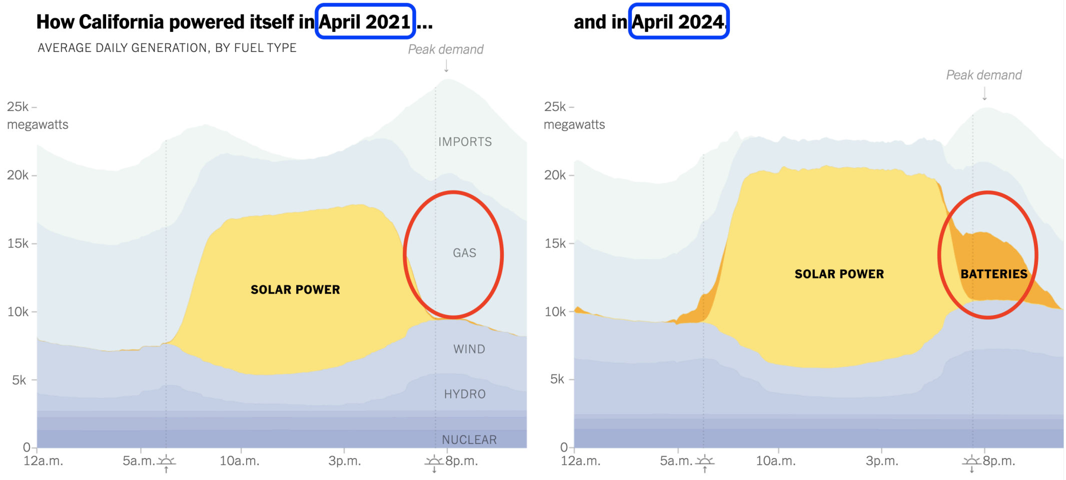


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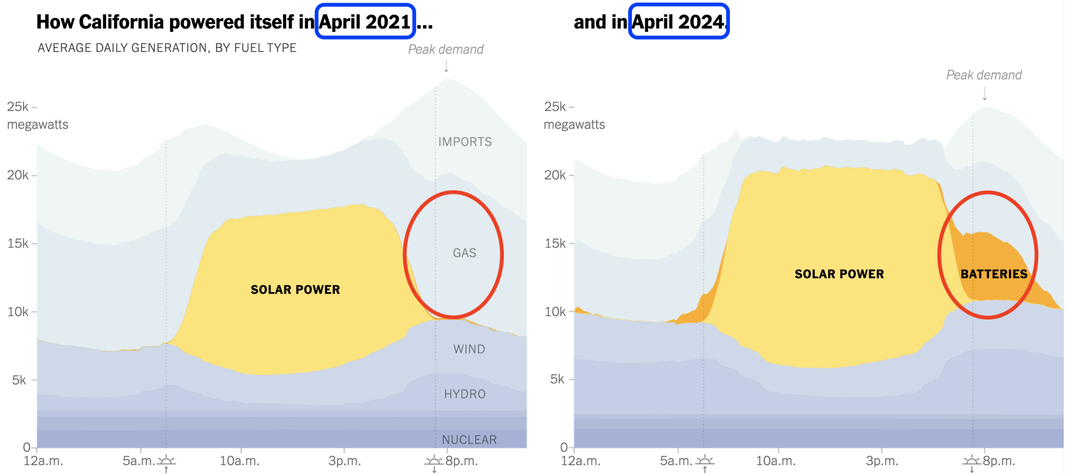


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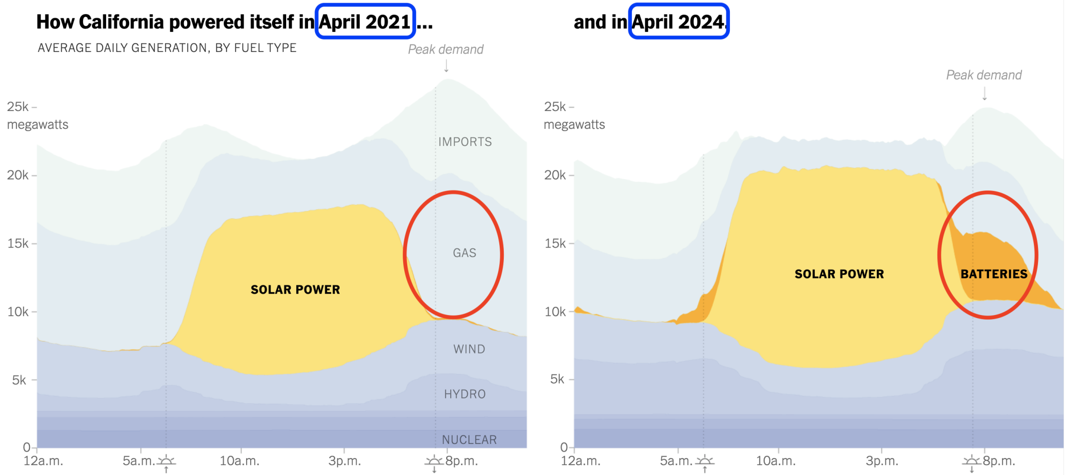


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# Regulators are already worried about battery market power

**CAISO, Storage Default Energy Bid Initiative**

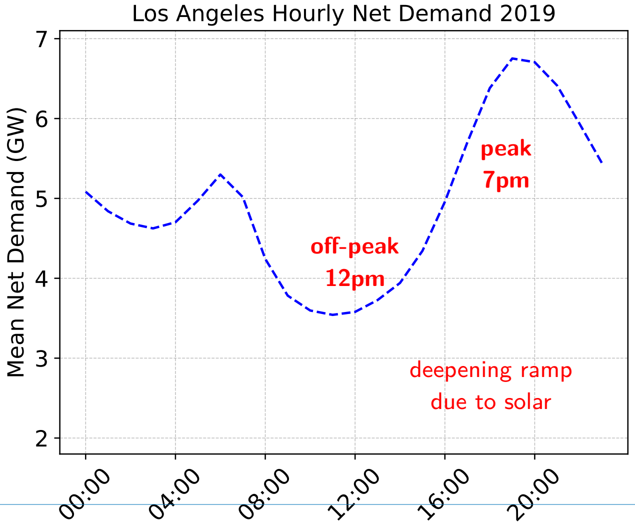
*"None of these storage resources are currently subject to market power mitigation, and the CAISO believes that it is important to develop mitigation measures to manage market power given the rapidly growing number and influence of energy storage resources."*

Battery market power is not just a theoretical concern; system operators are already thinking about mitigation.

Source: CAISO special report on battery storage

# The operational opportunity: intraday net-demand mismatch

$$\text{net demand} = \underbrace{\text{system demand}}_{\substack{\text{constant}\\ \text{year-on-year}}} - \underbrace{\text{renewables}}_{\substack{\text{increasing}\\ \text{year-on-year}}} = \text{dispatchable resources}$$



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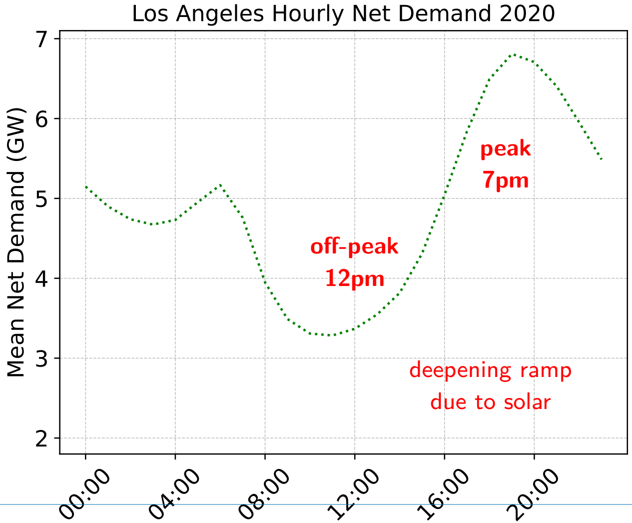

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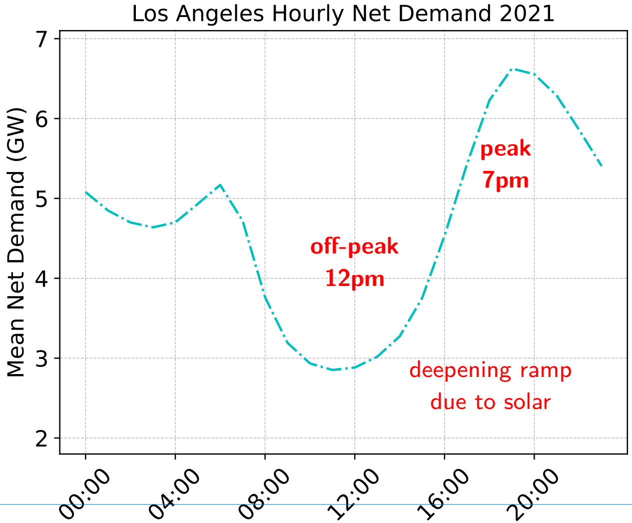

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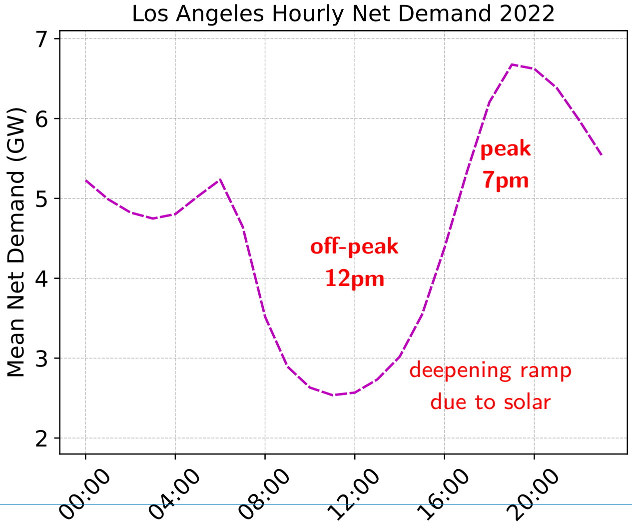

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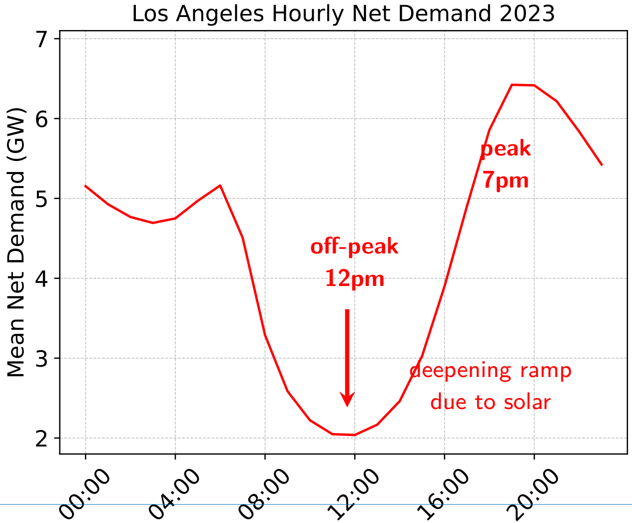

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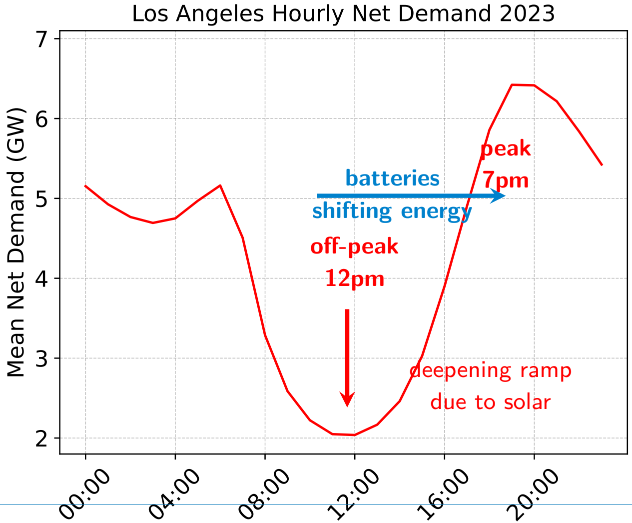

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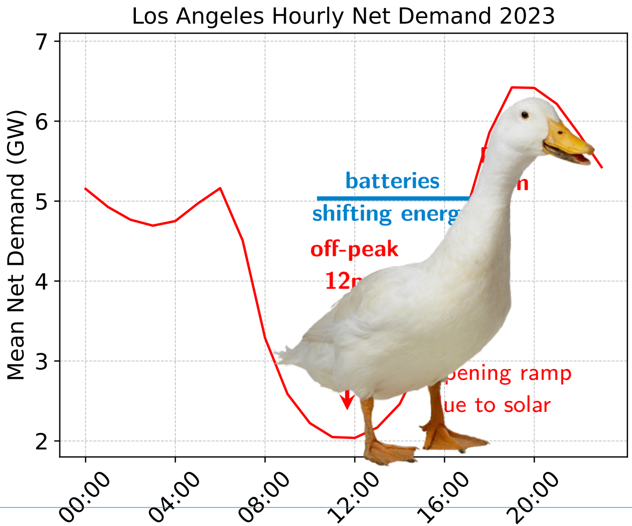
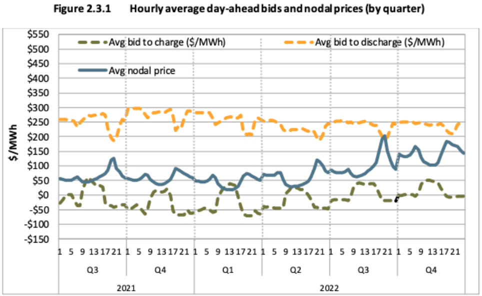
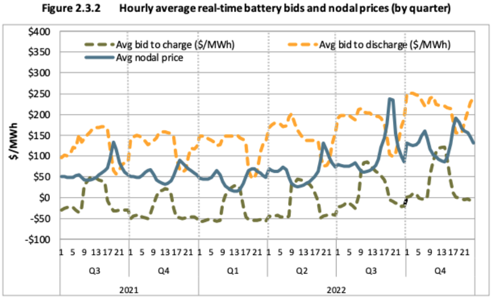


# Strategic behavior is not just a theoretical concern




Day-ahead: discharge bids often far above prices




Real-time: discharge bids move closer to prices

A visible distortion: batteries can avoid day-ahead scheduling and show up later in real time.

Source: CAISO special report on battery storage

# This paper in one slide

**Research question**

How does strategic battery operation distort day-ahead planning and real-time balancing relative to centralized operation?

**Model**

$T$-period two-settlement market

DA schedules + RT recourse

slow/fast generators

**Mechanism**

Quantity withholding

DA-to-RT shift

Weaker RT response

**Welfare**

$9/8 \leq \mathrm{PoA} \leq 4/3$

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# Model

# Two settlements: DA planning first, RT balancing later

**Day-Ahead Market (DA)** → **Real-Time Market (RT)**

**based on forecast**

**based on realized demand**

(1) forward market reduces uncertainty

(2) slow generators take time to start and ramp up

demand must equal supply at all times

**financially but not physically binding**

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# Model: a $T$-period two-settlement market

## Battery decisions

- DA schedule: $z_t^{\mathrm{DA}}$, chosen before the day.
- RT policy: $z_t^{\mathrm{RT}}(D_{1:t})$, chosen as demand is observed.

Positive $z$ means discharge; negative $z$ means charge.

## Demand process

$$(D_1, \dots, D_T) \sim \pi, \qquad \mu_t = \mathbb{E}[D_t].$$

Demand may be stochastic and correlated across hours.

## Energy balance

$$\sum_{t=1}^{T} z_t^{\mathrm{DA}} = 0, \qquad \sum_{t=1}^{T} z_t^{\mathrm{RT}}(D_{1:t}) = 0.$$

## Three regimes

- **No battery**
- **Centralized battery**: minimize system cost
- **Decentralized battery**: maximize battery profit

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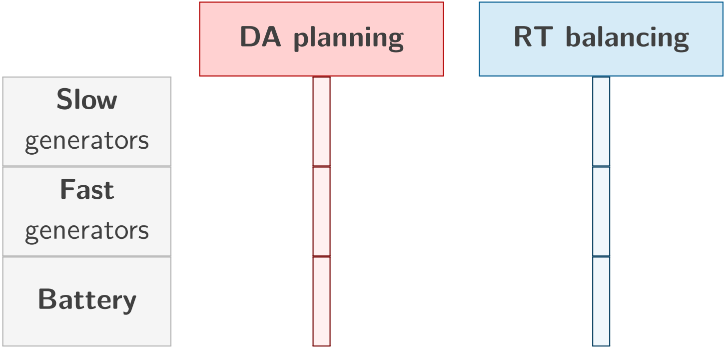

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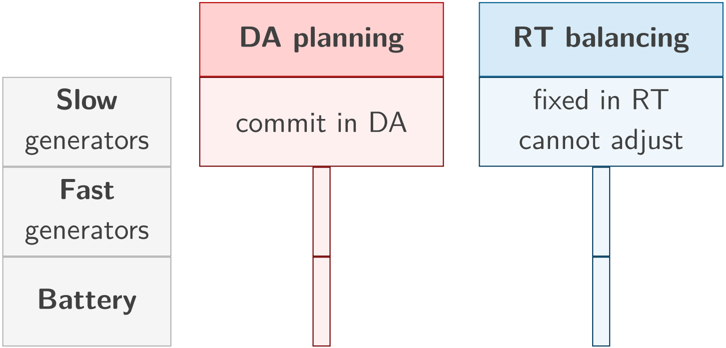

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### Day-ahead clearing

Forecasted net demand

$$\mu_t - z_t^{\mathrm{DA}}$$

is cleared by **slow + fast** supply.

### Real-time balancing

Realized imbalance

$$D_t - \mu_t - z_t^{\mathrm{RT}}(D_{1:t})$$

is cleared by **fast** supply only.

# Centralized battery: the smoothing benchmark

## Day-ahead smoothing

$$z_t^{\mathrm{DA,CN}} = \mu_t - \bar{\mu}, \qquad \mu_t - z_t^{\mathrm{DA,CN}} = \bar{\mu}.$$

The planner fully flattens predictable net demand.

## Real-time smoothing

At time $t$, the planner spreads the remaining imbalance over periods $t, \ldots, T$:

$$(D_t - \mu_t) - z_t^{\mathrm{RT,CN}}(D_{1:t}) = \frac{(D_t - \mu_t) + \sum_{i=t+1}^{T} (\mu_{i|D_{1:t}} - \mu_i) - B_t(D_{1:t-1})}{T - t + 1}.$$

Centralized operation smooths what is predictable and balances shocks as information arrives.

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# Mechanism

# Three mechanisms of battery market power

**1. Quantity withholding**

Price impact makes the battery move less energy.

**2. DA-to-RT shifting**

Sequential clearing lets the battery delay output into real time.

**3. Weak RT response**

Real-time price impact makes the battery respond less to demand shocks.

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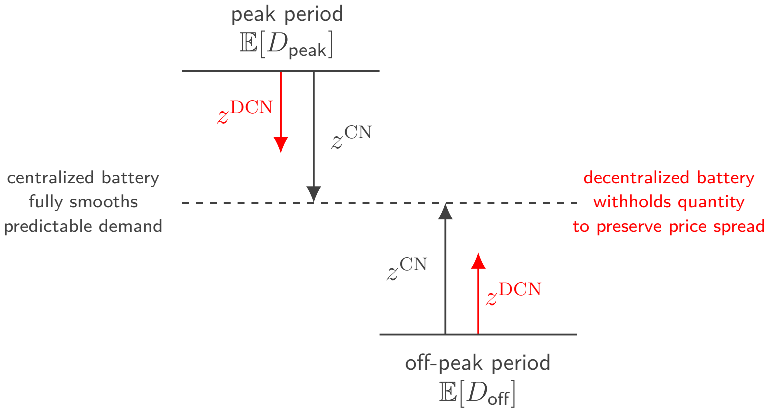
**Mechanism 1 intuition: the decentralized battery smooths less.**



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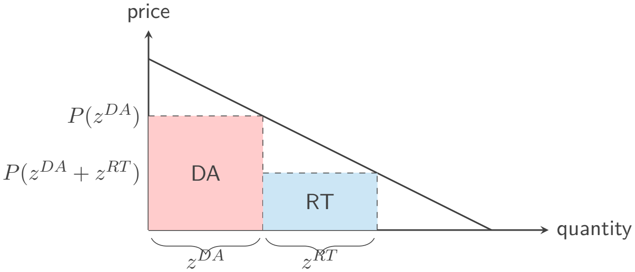
**Mechanism 2 intuition: the battery can hold back in DA and release more in RT.**



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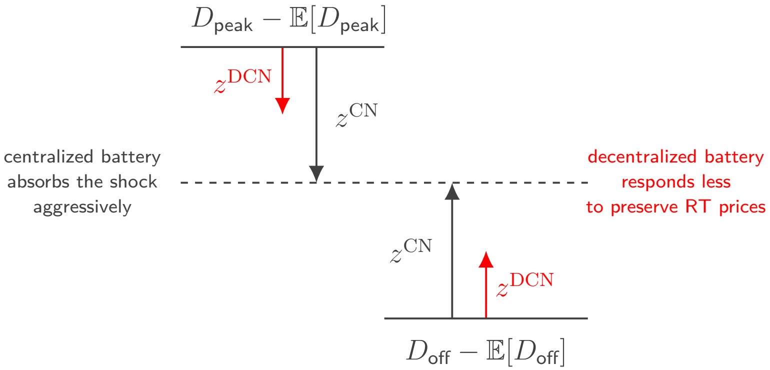
**Mechanism 3 intuition: in real time, the battery responds less to a demand surprise.**



# Distortion 1: quantity withholding

**Centralized**

Discharge until predictable net demand is flat. That equalizes prices and maximizes cost savings.

**Decentralized**

Discharging more lowers the price received on inframarginal units. The battery withholds to soften its own price impact.

**In the full model**

$$z_t^{\mathrm{DA,DCN}} + \mathbb{E}[z_t^{\mathrm{RT,DCN}}] = \frac{2}{4-k_f}(\mu_t - \bar{\mu}).$$

$$\text{withholding} = \frac{2-k_f}{4-k_f}.$$

50% when $k_f \approx 0$; 33% when $k_f \approx 1$.

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# Distortion 2: shifting participation from day-ahead to real-time

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$$\mathbb{E}[z_t^{\mathrm{RT,CN}}] = 0.$$

The planner uses DA for predictable smoothing and RT for surprises.

**Decentralized**

$$\mathbb{E}[z_t^{\mathrm{RT,DCN}}] = \frac{k_f}{4-k_f}(\mu_t - \bar{\mu}).$$

The battery splits predictable output across the two settlements.

Sequential markets let the battery reduce price impact by delaying some expected discharge into RT.

**RT share of expected discharge**

$$\frac{\mathbb{E}[z_t^{\mathrm{RT,DCN}}]}{z_t^{\mathrm{DA,DCN}} + \mathbb{E}[z_t^{\mathrm{RT,DCN}}]} = \frac{k_f}{2}.$$

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The planner uses DA for predictable smoothing and RT for surprises.

## Decentralized

$$\mathbb{E}[z_t^{\mathrm{RT,DCN}}] = \frac{k_f}{4 - k_f}(\mu_t - \bar{\mu}).$$

The battery splits predictable output across the two settlements.

Sequential markets let the battery reduce price impact by delaying some expected discharge into RT.

## RT share of expected discharge

$$\frac{\mathbb{E}[z_t^{\mathrm{RT,DCN}}]}{z_t^{\mathrm{DA,DCN}} + \mathbb{E}[z_t^{\mathrm{RT,DCN}}]} = \frac{k_f}{2}.$$

# Distortion 3: reduced real-time responsiveness

**The cleanest way to show the stochastic formula**

$$z_t^{\mathrm{RT,DCN}}(D_{1:t}) = \underbrace{\frac{k_f}{4-k_f}(\mu_t - \bar{\mu})}_{\text{predictable RT shift}} + \underbrace{\frac{1}{2} z_t^{\mathrm{RT,CN}}(D_{1:t})}_{\text{shock response}}.$$

- Centralized RT dispatch absorbs demand surprises efficiently.
- Decentralized RT dispatch behaves like Cournot supply of balancing service.
- Every shock term enters with coefficient 1/2.
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# Welfare and competition

## Cost ranking and Price of Anarchy

### Cost ranking

$$\mathrm{Cost}(\mathrm{NB}) \geq \mathrm{Cost}(\mathrm{DCN}) \geq \mathrm{Cost}(\mathrm{CN}).$$

### Price of Anarchy

$$\mathrm{PoA} = \frac{\mathrm{Cost}(\mathrm{NB}) - \mathrm{Cost}(\mathrm{CN})}{\mathrm{Cost}(\mathrm{NB}) - \mathrm{Cost}(\mathrm{DCN})}.$$

$\mathrm{PoA}$ compares cost savings from centralized operation to cost savings from decentralized operation.

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$\mathrm{PoA}$ compares cost savings from centralized operation to cost savings from decentralized operation.

# The monopoly loss is bounded

## Theorem

$$\frac{9}{8} \le \mathrm{PoA} \le \frac{4}{3}$$

Holds for any horizon and demand distribution.

## Meaning

Decentralized storage captures $75\%$–$88.9\%$ of the centralized cost reduction.

## Why the proof is nontrivial

The DA/RT cost has cross terms from forecast updates, real-time shocks, and the energy-balance constraint.

## Key move

Centralized real-time smoothing is orthogonal to the residual shock:

$$\sum_t \mathbb{E}\left[(D_t - \mu_t) z_t^{\mathrm{RT,CN}}\right] = \sum_t \mathbb{E}\left[(z_t^{\mathrm{RT,CN}})^2\right] \equiv S \ge 0.$$

So the same object is both a covariance and a squared norm.

## Consequence

All cost reductions collapse to the same two nonnegative ingredients:

$$A = T\sigma_\mu^2 \quad \text{and} \quad S \ge 0.$$

$$\Delta^{\mathrm{CN}} = a_A A + a_S S, \qquad \Delta^{\mathrm{DCN}} = b_A A + b_S S.$$

Thus $\mathrm{PoA} = \Delta^{\mathrm{CN}}/\Delta^{\mathrm{DCN}}$ is bounded by the two coefficient ratios.

# Impact of Competition

We now consider $n$ big batteries in Cournot competition.

We can derive battery strategies in closed form.

**Theorem**

$$1 + \frac{1}{n(n+1)(n^2+n+2)} \leq \mathrm{PoA} \leq 1 + \frac{1}{n(n+2)}.$$

The bounds are tight, and $\mathrm{PoA}$ decreases in $k_f$.

$\mathrm{PoA} - 1 = O(1/n^2)$ worst case, $O(1/n^4)$ best case.

Competition is very effective at aligning incentives.

*Caveat: battery profit is reduced, which may discourage entry.*

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# Mitigation can backfire

## Natural interventions

1. Require expected RT discharge to be zero.
2. Add virtual bidders to compress DA/RT price gaps.
3. Subsidize discharge in target periods.

## What happens

- visible DA-to-RT shifting falls
- quantity withholding rises
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## Operational frictions

- battery inefficiency
- ramping costs
- alternative balance constraints
- homogeneous capacity constraints
- battery investment and operations

## Market structure

- non-parallel linear supply curves
- strategic generators
- heterogeneous capacity constraints†
- convex inverse supply curves*

## Same upper bound continues to hold

$$\mathrm{PoA} \leq 1 + \frac{1}{n(n+2)}.$$

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# Convex inverse supply curves

| | $R=1$ | $R=2$ | $R=3$ | $R=4$ | $R=5$ |
|---|---|---|---|---|---|
| $n=1$ | 1.333 | 1.421 | 1.463 | 1.487 | 1.504 |
| $n=2$ | 1.125 | 1.143 | 1.149 | 1.152 | 1.153 |
| $n=3$ | 1.067 | 1.076 | 1.080 | 1.081 | 1.082 |
| $n=4$ | 1.042 | 1.049 | 1.054 | 1.057 | 1.058 |
| $n=5$ | 1.029 | 1.035 | 1.039 | 1.042 | 1.044 |

Tight upper bounds from Table 2 for $p(q) = \alpha + \beta q^R$.

# Calibration

# Calibration: California and Texas

**Data**

Hourly day-ahead and real-time prices and net demand in CAISO and ERCOT.

**Calibrating real-time flexibility**

Use price–net-demand regressions to estimate how much supply responds in real time:

$$\hat{k}_f = \frac{\text{predictable-demand slope}}{\text{real-time-surprise slope}}.$$

| | California, $\hat{k}_f = 0.81$ | | | | | Texas, $\hat{k}_f = 0.56$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| PoA | 1.201 | 1.058 | 1.027 | 1.016 | 1.010 | 1.284 | 1.100 | 1.052 | 1.032 | 1.021 |

Competition sharply reduces calibrated inefficiency in both California and Texas.

# Conclusion

## Main takeaways

Strategic batteries distort storage value along three margins:

| **Withhold** | **Delay** | **Under-respond** |
|---|---|---|
| less total discharge | more RT participation | weaker RT balancing |

The welfare loss is real, bounded, and rapidly disciplined by competition.

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# Market Design = Strategic Behavior + Optimization

### Energy

"Battery Operations in Electricity Markets" (**JMP**)

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### Information Design: Social Services + Queues

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# Future Work

## Energy Markets
(use existing things well)

Gridmatic

industry collaboration

battery bidding

transmission networks

efficient algorithms

## Energy Ecosystem
(make new things happen)

BASE

new business & financing models

demand flexibility

renewable sourcing

## Fundamental Theory
(motivated by applications)

robust multi-unit auctions

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# Contract Design and Operations for Residential Battery Programs

### Motivation

Outages are becoming more frequent and longer, while grid upgrades are slow and costly. Residential batteries can provide both *household backup* and *grid support*.

### Operational problem

During normal periods, enrolled batteries can reduce peak generation costs. But stored energy also has future value for reliability during outages.

### Program design

The utility offers a menu: no battery, home-backup-only battery, or utility-operated battery. Incentives determine which households enroll.

### Modeling idea

Jointly optimize participation incentives and battery dispatch. The optimal operating rule has a simple two-threshold structure.

The utility must design incentives and operations together: backup reliability changes the value of stored energy.

# Residential Battery Pooling Under Backup Commitments

## Motivation

Home batteries are *dual-use*: market assets and household backup products.

## Question

What value does fleet coordination create when each home keeps its own backup promise?

## Modeling idea

Compare standalone vs. pooled MPC. Pooling allows energy sharing, but each home retains its own battery state and reserve floor.

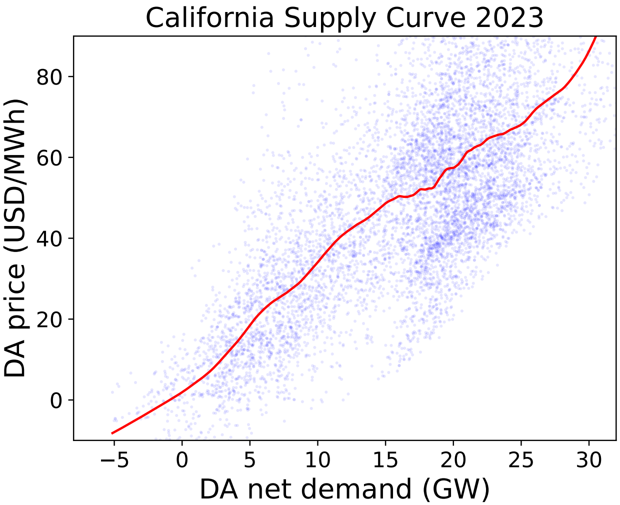
## Empirical setting

**543 homes**, ERCOT prices, 15-minute control. Outcome: firm margin while preserving backup commitments.

Pooling is valuable, but not a virtual-battery abstraction: resilience promises remain household-specific.

# Backup

# California's inverse-supply curve is ≈ linear



# Backup: heterogeneous battery capacity

## General capacity bound

Let $C_b$ be battery $b$'s capacity, $C_{\text{tot}} = \sum_{j=1}^{n} C_j$, and $w_b = C_b / C_{\text{tot}}$. Then

$$\text{PoA} \leq \frac{2n+3}{2n+4} + \frac{1}{2} \sum_{b=1}^{n} w_b^2.$$

## Identical capacities

If $C_1 = \cdots = C_n$, then $\sum_b w_b^2 = 1/n$, so

$$\text{PoA} \leq 1 + \frac{1}{n(n+2)}.$$

## Dominant battery

If one battery's capacity share tends to one, the outcome converges to the monopoly benchmark:

$$\text{PoA} \to \text{PoA}^{(1)} \leq \frac{4}{3}.$$

Capacity concentration moves the system from the competitive bound toward the monopoly bound.

# Backup: full centralized RT decomposition

At time $t$, define remaining RT balance obligation

$$B_t(D_{1:t-1}) = -\sum_{s=1}^{t-1} z_s^{\mathrm{RT,CN}}(D_{1:s}).$$

Then

$$z_t^{\mathrm{RT,CN}}(D_{1:t}) = \frac{T-t}{T-t+1}(D_t-\mu_t) - \frac{1}{T-t+1}\sum_{i=t+1}^{T}(\mu_{i|D_{1:t}}-\mu_i) + \frac{1}{T-t+1}B_t(D_{1:t-1}).$$

- current surprise term;
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$$B_t(D_{1:t-1}) = -\sum_{s=1}^{t-1} z_s^{\mathrm{RT,CN}}(D_{1:s}).$$

Then

$$z_t^{\mathrm{RT,CN}}(D_{1:t}) = \frac{T-t}{T-t+1}(D_t - \mu_t) - \frac{1}{T-t+1}\sum_{i=t+1}^{T}(\mu_{i|D_{1:t}} - \mu_i) + \frac{1}{T-t+1}B_t(D_{1:t-1}).$$

- current surprise term;
- correlation / forecast-update term;
- remaining balance term.

# Backup: no-battery and centralized costs

## No battery

$$\mathrm{Cost}(\mathrm{NB}) = \sum_{t=1}^{T} \left( \alpha \mu_t + \frac{\beta}{2} \mu_t^2 + \frac{\beta}{2k_f} \mathsf{Var}(D_t) \right).$$

## Centralized cost reduction

Let $\sigma_\mu^2 = T^{-1} \sum_t (\mu_t - \bar{\mu})^2$ and $S = \sum_t \mathbb{E}[(D_t - \mu_t) z_t^{\mathrm{RT,CN}}]$. Then

$$\mathrm{Cost}(\mathrm{NB}) - \mathrm{Cost}(\mathrm{CN}) = \beta \left( \frac{1}{2} T \sigma_\mu^2 + \frac{1}{2k_f} S \right).$$

# Backup: decentralized cost reduction

With the same notation,

$$\mathrm{Cost}(\mathrm{NB}) - \mathrm{Cost}(\mathrm{DCN}) = \beta\left(\frac{12 - 5k_f + k_f^2}{2(4-k_f)^2} T\sigma_\mu^2 + \frac{3}{8k_f} S\right).$$

## Proof idea for the PoA bounds

The numerator and denominator are positive linear combinations of the same two nonnegative objects, $T\sigma_\mu^2$ and $S$. The coefficient ratios are bounded between $9/8$ and $4/3$.

# Backup: $n$ Cournot batteries in closed form

For each battery,

$$z_{b,t}^{\mathrm{DA,DCN}} = \frac{n+1-k_f}{(n+1)^2 - nk_f}(\mu_t - \bar{\mu}),$$

$$z_{b,t}^{\mathrm{RT,DCN}} = \frac{k_f}{(n+1)^2 - nk_f}(\mu_t - \bar{\mu}) + \frac{1}{n+1} z_t^{\mathrm{RT,CN}}.$$

**Interpretation**

Each battery internalizes only part of the price impact, so the strategic distortions vanish as $n$ grows.

# Backup: policy substitution logic

**Policy tries to reduce**

DA-to-RT shift

**Battery substitutes toward**

Quantity withholding

A mitigation rule can reduce the most visible distortion while increasing the more costly one.