Robust Auctions with Support Information

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Motivation

- mech design: how to optimally sell things
- classical theory too detail-dependent, so we **relax the common prior assumption** (Wilson doctrine)

Problem Formulation

- Optimize over direct mechanisms (x, p) selling one indivisible item to n buyers.
- mechanism is **prior-independent**
  - no need to know F ("detail-free" or "robust")
  - performance guarantee over all $F \in \mathcal{F}$
- Only the **upper bound** b and **lower bound** a are assumed known.
- We consider many dist classes on $[a, b]^n$.
- **dominant strategy** IC+IR
  - each buyer need not know other buyers' dists
- Objective = "λ-regret" on revenue (unifies *regret* and *ratio* objectives)
  - minimax regret means $\lambda = 1$
  - maximin ratio = λ s.t. minimax λ-regret is zero
- Benchmark = maximum possible revenue when valuation is known = $\max(\boldsymbol{v})$.

Research question:
What is an optimal detail-free mechanism and how well can we perform?

Challenges

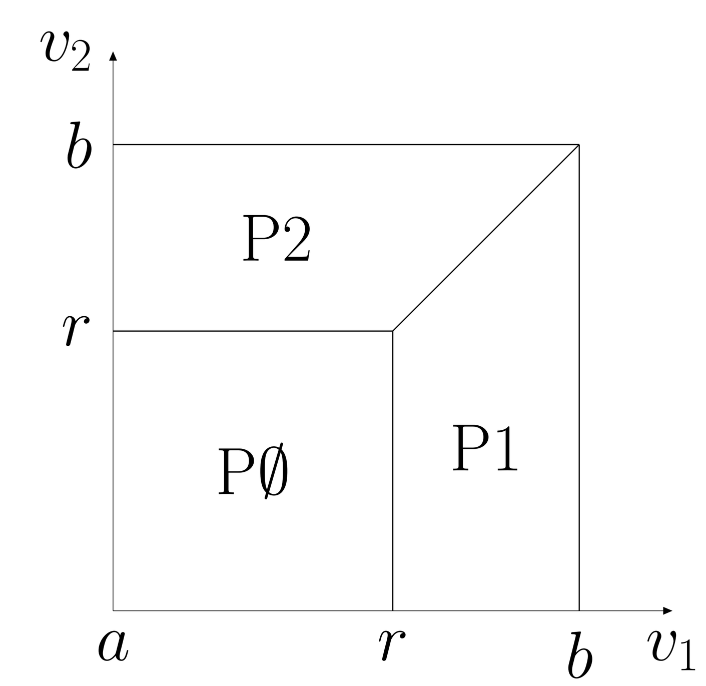
Minimax problems are hard!
Our problem is **nonconvex** due to class restriction in $\mathcal{F}$ e.g. i.i.d.

Minimax Problem For Each Distribution Class $\mathcal{F}$

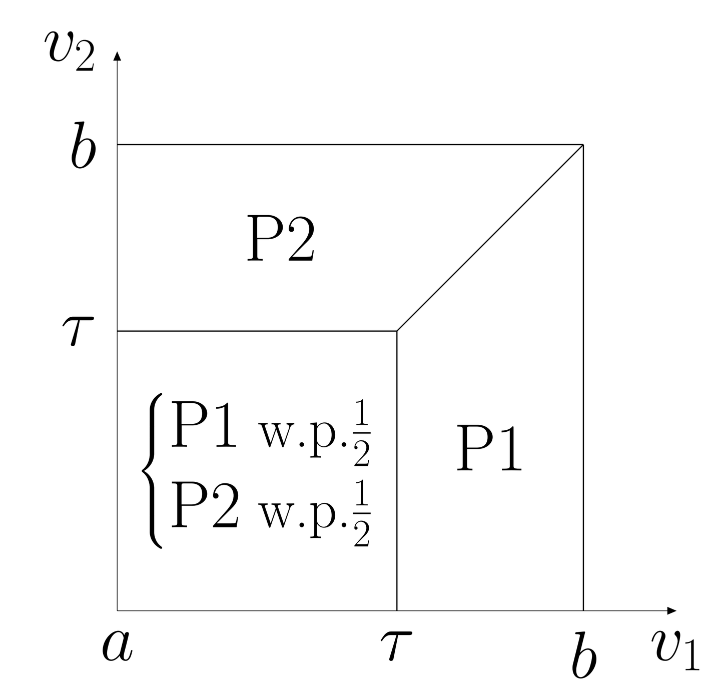
$$\min_{\substack{\text{mech } (x,p)\in\mathcal{M} \\ \text{IC+IR}}} \max_{\boldsymbol{F}\in\mathcal{F}} \mathbb{E}_{\boldsymbol{v}\sim\boldsymbol{F}}\left[\lambda \underbrace{\max(\boldsymbol{v})}_{\text{benchmark}} - \underbrace{\sum_{i=1}^{n} p_i(\boldsymbol{v})}_{\text{revenue}}\right]$$

Main Result: Optimal Mechanism Classes SPA and POOL

We introduce the POOL(τ) "pooling auction" mechanism class!



(a) SPA(r) allocation rule
always allocates to the highest-value agent if the highest value is above r; otherwise, does not allocate



(b) POOL(τ) allocation rule
always allocates to the highest-value agent if the highest value is above τ; otherwise, allocates to each one of the n agents uniformly at random w.p. $1/n$.

Main Theorem

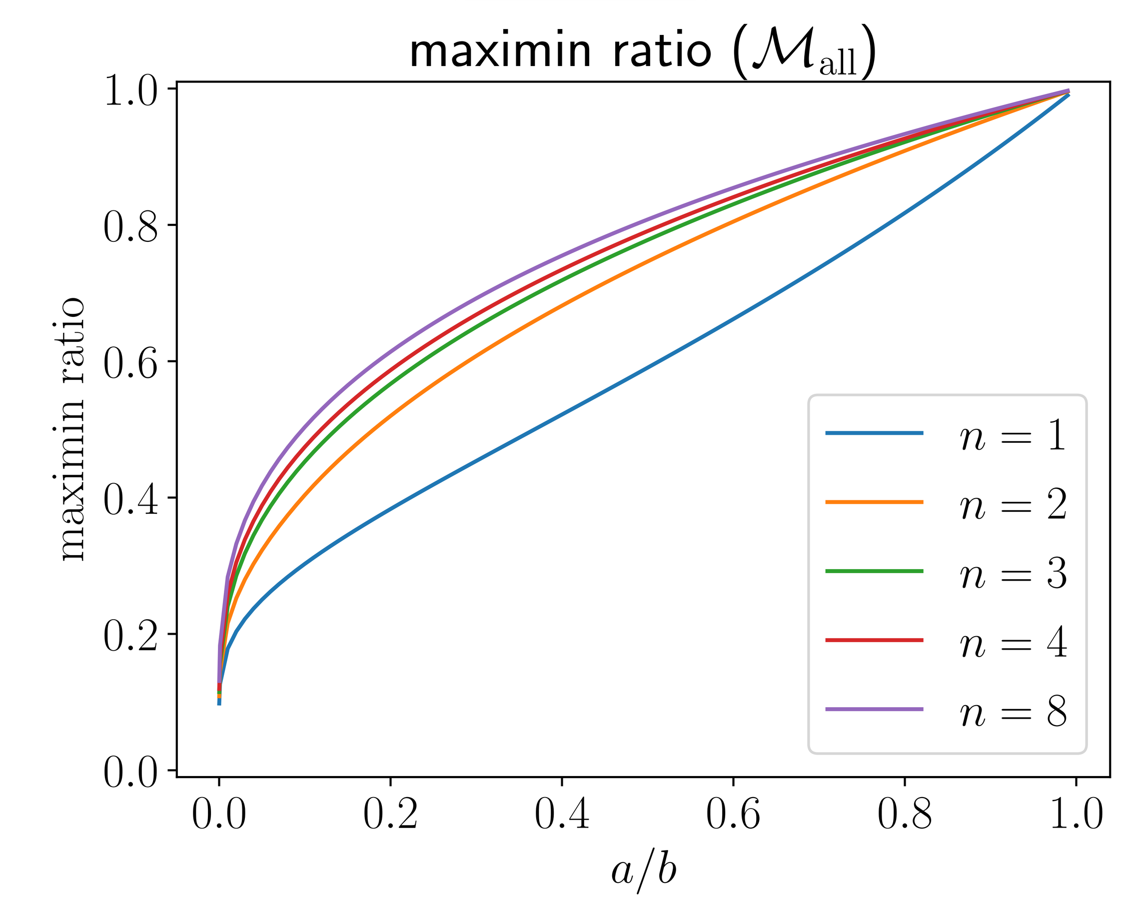
For any $\lambda \in [0, 1]$ and $n \geq 1$ i.i.d. bidders, there are constants $k_l < k_h$ s.t. the optimal minimax λ-regret mechanism m^* depends on the *relative support information* a/b as:

low information	moderate information	high information
$a/b \leq k_l$	$k_l \leq a/b \leq k_h$	$a/b \geq k_h$
$m^* = \text{SPA}(\Phi)$	$m^* = \begin{cases}\text{SPA on } [a, v^*] \\ \text{POOL on } [v^*, b]\end{cases}$	$m^* = \text{POOL}(\Psi)$

Our Approach

- **saddle point approach**: find m^*, F^*
  $R(m^*, \boldsymbol{F}) \leq R(m^*, \boldsymbol{F}^*) \leq R(m, \boldsymbol{F}^*)\ \forall m, \boldsymbol{F}$
- Nature's saddle: with our mechanism forms (SPA and POOL), regret depends only on the marginal $F(v)$ pointwise – so pointwise optimization works!
- Seller's saddle: Bayesian mech design.

Quantitative Insights



Separation between mechanism classes $\mathcal{M}$ quantify the power of *mechanism features*: